Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-11

Problem 1. Let $z = f(x, y) = \sqrt{1 + xy}$. Rewrite z in terms of r, θ and compute $\partial z/\partial r$ and $\partial z/\partial \theta$ when $x = 6, y = 8, r = 10$. (Later on we will see how to do this using the chain rule.)

Let $\mathbf{u}$ be the unit vector $\langle 3/5, 4/5 \rangle$. Compute $D_{\mathbf{u}} f(6, 8) = \langle f_x(6, 8), f_y(6, 8) \rangle \cdot \mathbf{u}$.

$$z = \sqrt{1 + r^2 \sin\theta \cos\theta}$$

Note: $\cos\theta = \frac{x}{r} = 3/5$

$\sin\theta = \frac{y}{r} = 4/5$

$$\frac{\partial z}{\partial r} = \frac{1}{2}(1 + r^2 \sin\theta\cos\theta)^{-1/2}\, 2r\sin\theta\cos\theta$$

$$= \cancel{\tfrac{1}{2}}(49)^{-1/2} \cdot \cancel{2} \cdot 8 \cdot 3/5 = \boxed{\frac{24}{35}}$$

$$\frac{\partial z}{\partial \theta} = \ldots\ldots \text{similar work} = \boxed{-2}$$

$$D_{\vec{u}} f(6,8) = \left\langle \frac{1}{2}(1 + 6\cdot 8)^{-\frac{1}{2}} \cdot 8,\ \frac{1}{2}(1 + 6 \cdot 8)^{-\frac{1}{2}} \cdot 6 \right\rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$$

$$= \left\langle \frac{4}{7}, \frac{3}{7} \right\rangle \cdot \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle = \boxed{\frac{24}{35}}$$

Problem 2. Consider the equation $yz + x\ln y + z^3 = 0$. This equation implicitly defines $z = f(x,y)$ as a function of x, y. Compute $f_x(3,1)$ and $f_y(3,1)$.

Treat y as const. & diff w.r.t. x:

$$y\frac{\partial z}{\partial x} + \ln y + 3z^2\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{-\ln y}{y+3z^2} = \boxed{0}$$

at $(3,1)$

Similarly: treat x as const. & diff w.r.t. y:

$$z + y\frac{\partial z}{\partial y} + \frac{x}{y} + 3z^2\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = \frac{-z - x/y}{y+3z^2} = \frac{-0-3/1}{1+0} = \boxed{-3}$$

at $(3,1)$

$x=3$, $y=1$, so to find z:

$$1z + 3\ln(1) + z^3 = 0$$

$$z + z^3 = 0$$

$$z = 0.$$